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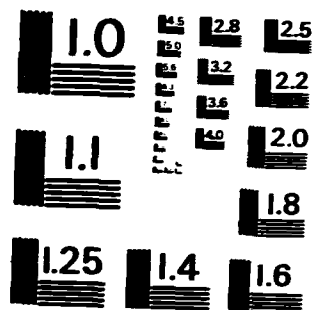
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A CALCULATION OF WRINKLED FLAMES

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A CALCULATION OF WRINKLED FLAMES

H. V. McConnaughey, G. S. S. Ludford and G. I. Sivashinsky

Technical Summary Report #2480

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ABSTRACT

In an earlier work, Sivashinsky derived a nonlinear integro-differential equation for spontaneous hydrodynamic instability of a plane flame front. The simplified form of that equation which describes a steady progressive wave of long period is reconsidered here. It is shown to represent a wrinkled flame front whose slope has a logarithmic singularity at each wrinkle. We have computed the flame profile and its speed of propagation more accurately than done previously by Michelson and Sivashinsky, and find that the increase in propagation speed due to wrinkling is in surprisingly good agreement with recent experimental findings.

AMS (MOS) Subject Classifications: 80A25, 76E99, 45Q05

Key Words: hydrodynamic instability, wrinkled flame, steady progressive wave, logarithmic singularity.

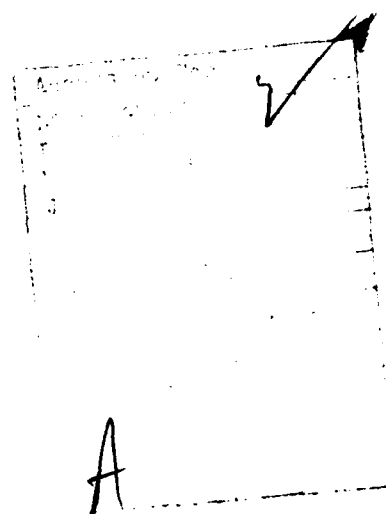
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SIGNIFICANCE AND EXPLANATION

Spontaneous instability of laminar flames has been observed in many experiments. A primary destabilizing effect is known to be thermal expansion of the gas passing through the flame front and causing hydrodynamic instability of the flame. This is manifested by a wrinkling of the flame and the resulting corrugated flame front may be seen to propagate at a constant velocity with its wrinkled shape well preserved. The speed of propagation of the wrinkled flame is markedly greater than that of the undisturbed flame.

This paper demonstrates the above-described phenomenon through theoretical means. The nonlinear integro-differential evolution equation derived by Sivashinsky for spontaneous hydrodynamic instability of a laminar flame is considered. A steady progressive long-wave solution of that equation is shown to produce a wrinkled flame front with a logarithmic singularity in slope at the cusp of each wrinkle. The calculated increase in propagation speed is found to agree well with experimental observation.



The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the authors of this report.

A CALCULATION OF WRINKLED FLAMES

H. V. McConnaughey, G. S. S. Ludford and G. I. Sivashinsky

Introduction

A nonlinear flame stability analysis carried out by Sivashinsky (1977) considers instability of a laminar plane flame to disturbances in the hydrodynamic field. In that work, a nonlinear integro-differential equation describing the evolution of the perturbed flame front is derived. The present study seeks a solution of that equation which represents the experimentally observed phenomenon of a steadily propagating flame with corrugated or "wrinkled" profile (Markstein, 1970; Lind and Whitson, 1977; Ivashchenko and Rumyantsev, 1978; Groff, 1982). Attention is accordingly focused on the simplified equation

$$\frac{1}{2} \bar{F}_0^2(\eta) + \gamma \int_{-\infty}^{\infty} \frac{\bar{F}_0(\bar{\eta})}{\bar{\eta} - \eta} d\bar{\eta} = v, \quad (1)$$

which will be motivated presently. The function $\bar{F}_0(\eta)$ is the shape of the disturbed flame front which has been assumed to be a steady progressive wave of long (but arbitrary) period; v is the constant speed of propagation of the front; η is the space coordinate. These quantities are all dimensionless. The parameter γ is proportional to

$$\frac{\rho_u}{\rho_b} - 1,$$

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where ρ_b/ρ_u is the ratio of the density of the burned gas to that of the unburned gas (the thermal expansion coefficient).

It is found that equation (1) does indeed describe a wrinkled flame front. The purpose of this work is to show that the slope of the flame profile has a logarithmic singularity at the cusp of each wrinkle and to give an accurate calculation of the profile.

Development and Discussion of Equations

Equation (1) is obtained from the more general equation

$$F_{0\tau} + 4F_{0\eta\eta\eta\eta} \pm \frac{1}{l_s} F_{0\eta\eta} + \frac{1}{2} F_{0\eta}^2 + \gamma \int_{-\infty}^{\infty} \frac{F_{\bar{\eta}}(\bar{\eta}, \tau)}{\bar{\eta} - \eta} d\bar{\eta} = 0 \quad (2)$$

Here, $F_0(\eta, \tau)$ is the perturbed flame front; η and τ are the space and time variables; l_s is a positive $O(1)$ constant which plays an unimportant role in the present discussion. For a derivation of (2), the reader is referred to Buckmaster and Ludford (1982). Equation (2) is seen to be equivalent to the aforementioned equation derived by Sivashinsky (1977) by recognizing the identity

$$\int_{-\infty}^{\infty} |k| \exp[ik(\eta - \bar{\eta})] F_0(\bar{\eta}, \tau) d\bar{\eta} dk = 2 \int_{-\infty}^{\infty} \frac{F_{\bar{\eta}}(\bar{\eta}, \tau)}{\bar{\eta} - \eta} d\bar{\eta}$$

(Buckmaster and Ludford, 1982).

The limit $\gamma \rightarrow 0$ and the slow scales $\gamma\eta$ and $\gamma^2\tau$ are now considered. Consequently, the fourth derivative term drops out of (2). Singling out the negative coefficient of $F_{0\eta\eta}$ then yields

$$F_{0\tau} - \frac{1}{l_s} F_{0\eta\eta} + \frac{1}{2} F_{0\eta}^2 + \gamma \int_{-\infty}^{\infty} \frac{F_{\bar{\eta}}(\bar{\eta}, \tau)}{\bar{\eta} - \eta} d\bar{\eta} = 0 \quad (3)$$

Michelson and Sivashinsky (1977) approximate the solution of this equation using a finite difference scheme. Their numerical work shows that an imposed

initial disturbance of large characteristic dimension gives rise to a steady progressive wrinkled flame front of the same characteristic length.

Information about the steady state can be extracted from (3) by analytical considerations. If $F_0(\eta, \tau)$ is taken to be a steady progressive wave, it may be written

$$F_0(\eta, \tau) = -V\tau + \bar{F}_0(\eta) ,$$

where V is a constant. In addition, for a wave of long period, the second derivative term may be neglected. The omission of $F_{0\eta\eta}$ is expected to be invalid, however, in the neighborhood of anticipated cusps in the flame profile where the curvature must be comparatively large in order to effect the sudden change in slope.

These modifications of (3) yield equation (1). Next, the origin is fixed at a point where $\bar{F}_{0\eta}$ vanishes, consequently V may be expressed as

$$V = \gamma \int_{-\infty}^{\infty} \frac{\bar{F}_{0\eta}(\bar{\eta})}{\bar{\eta}} d\bar{\eta} . \quad (4)$$

Substituting (4) into (1) gives the integral equation

$$\frac{1}{2} y^2(\eta) + \int_{-\infty}^{\infty} \left(\frac{1}{\eta - \bar{\eta}} - \frac{1}{\bar{\eta}} \right) y(\bar{\eta}) d\bar{\eta} = 0 , \quad (5)$$

where

$$y(\eta) = \gamma^{-1} \bar{F}_{0\eta}(\eta) .$$

A periodic solution of this equation is sought and, without loss of generality, the period is taken to be 2π . (A solution with arbitrary period $2p$ can then be obtained by changing the argument from η to $p\eta/\pi$; the integral in (4) shows that V is unaltered. Hence, all periodic profiles propagate at the same speed and have the same amplitude; only their wavelengths are different.)

Equation (5) may be rewritten as

$$\frac{1}{2} y^2(\eta) = - \int_{-\infty}^{\infty} y_{(2n-1)\pi}^{(2n+1)\pi} \left(\frac{1}{\eta-\eta} - \frac{1}{\eta} \right) y(\bar{\eta}) d\bar{\eta} ,$$

when $y(\eta)$ has period 2π , this is equivalent to

$$\begin{aligned} \frac{1}{2} y^2(\eta) &= - \int_{-\pi}^{\pi} y_{-\pi}^{\pi} \left(\frac{1}{\xi+2n\pi-\eta} + \frac{1}{\xi+2n\pi} \right) y(\xi) d\xi \\ &= - \frac{1}{2} y_{-\pi}^{\pi} \left[\cot\left(\frac{\xi-\eta}{2}\right) - \cot \frac{\xi}{2} \right] y(\xi) d\xi . \end{aligned} \quad (6)$$

The principal-value integral in (6) has singularities at $\eta = \pm\pi$, where

$$y^2(\eta) \sim y_{-\pi}^{\pi} \cot\left(\frac{\xi-\eta}{2}\right) y(\xi) d\xi . \quad (7)$$

The singular behavior there is found to be

$$y(\eta) \sim 2 \operatorname{sgn}(\eta\mp\pi) \ln|\eta\mp\pi| \quad \text{near } \eta = \pm\pi . \quad (8)$$

To see this at $\eta = \pi$, first note that

$$\begin{aligned} y_{-\pi}^{\pi} \cot\left(\frac{\xi-\eta}{2}\right) y(\xi) d\xi &= y_0^{2\pi} \cot\left(\frac{\xi-\eta}{2}\right) y(\xi) d\xi \\ &\sim y_0^{2\pi} \frac{2}{\xi-\eta} y(\xi) d\xi \quad \text{near } \eta = \xi = \pi . \end{aligned}$$

It is easily found that

$$y_0^{2\pi} \frac{4 \operatorname{sgn}(\xi-\pi) \ln|\xi-\pi|}{\xi-\eta} d\xi \sim 4 \ln^2|\eta-\pi| \quad \text{near } \eta = \pi ,$$

hence relation (8) satisfies relation (7) for $\eta \approx \pi$; the result for $\eta \approx -\pi$ follows similarly. The function $y(\eta)$ is thus seen to have a logarithmic singularity at odd factors of π , i.e., at the cusps of the flame front.

The singular nature at the cusps of the flame profile is reflected by the pertinent numerical results of Michelson and Sivashinsky (1977). They took the term $F_{0\eta\eta}$ (see equation (3)) into consideration in their calculations and found the curvature of the flame to be surprisingly high at the wrinkles. The singularity there creates a stagnation zone near each cusp in

the burned gas region, as occurs at the tip of a Bunsen flame (Figures 1a,b). The structure of the flame front at a wrinkle (Figure 1a), however, is quite different from that of a Bunsen wedge flame (Figure 1b).

Assume now that the wrinkled flame profile is symmetric, i.e., that $y(\eta)$ is an odd function. Equation (6) may then be written

$$y^2(\eta) = - \int_0^\pi \left[\cot\left(\frac{\xi-\eta}{2}\right) + \cot\left(\frac{\xi+\eta}{2}\right) - 2 \cot \frac{\xi}{2} \right] y(\xi) d\xi . \quad (9)$$

This equation describes an odd and 2π -periodic solution of equation (1). It is a simple exercise to show that equation (9) therefore represents the solution of the problem addressed by Sivashinsky (1976) in his discussion of corrugated flame fronts. In that analysis, the following equation for the shape of a two-dimensional, long-wave corrugated profile is derived

$$\lambda - \frac{1}{2} G^2(\eta) = \frac{1}{2\pi} \sum_{n=1}^{\infty} \int_{-\pi}^{\pi} G(\zeta) \sin n\zeta \cos n\eta d\zeta . \quad (10)$$

Here, the function $G(\eta)$, which is proportional to the slope of the flame front, is also assumed to be 2π -periodic and odd. The constant λ measures the speed of propagation of the front. This equation is recovered from (1) by writing $\bar{F}_{0\eta}(\bar{\eta})$ as the Fourier sine series

$$\frac{1}{\pi} \sum_{n=1}^{\infty} \sin n\bar{\eta} \int_{-\pi}^{\pi} \bar{F}_{0\zeta}(\zeta) \sin n\zeta d\zeta ,$$

so that the integral in (1) may be written

$$\sum_{n=1}^{\infty} \int_{-\pi}^{\pi} \bar{F}_{0\zeta}(\zeta) \sin n\zeta d\zeta \int_{-\pi}^{\pi} \frac{\sin n\bar{\eta}}{\bar{\eta}-\eta} d\bar{\eta} . \quad (11)$$

The principal-value integral in (11) is equal to $\pi \cos n\eta$. Hence, equation (10) is obtained by defining $G(\eta) = \bar{F}_{0\eta}(\eta)/2\pi$ and $\lambda = V/(2\pi)^2$.

In the analysis of Sivashinsky (1976), the solution of (10) is crudely approximated by the solution of

$$\lambda - \frac{1}{2} G^2(\eta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} G(\zeta) \sin \zeta \cos \eta d\zeta .$$

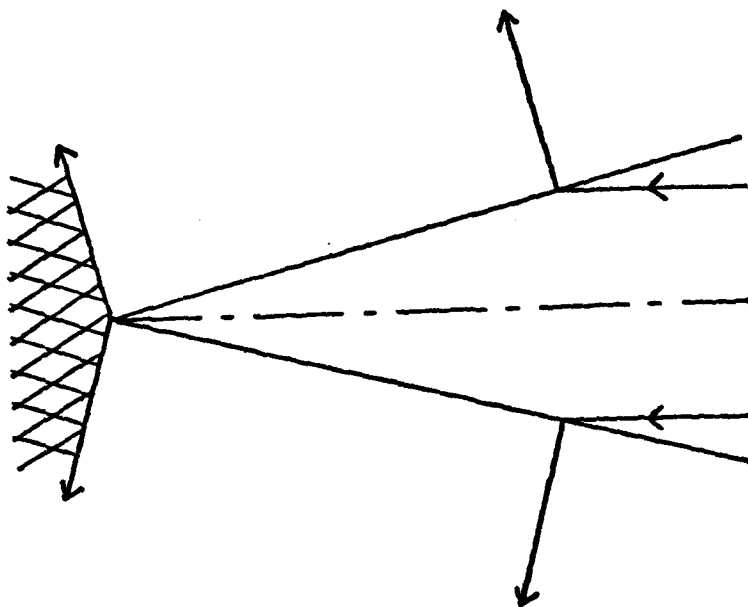


Figure 1b

Cusp of wrinkled flame and tip of Bunsen flame. Shaded regions are stagnation zones.

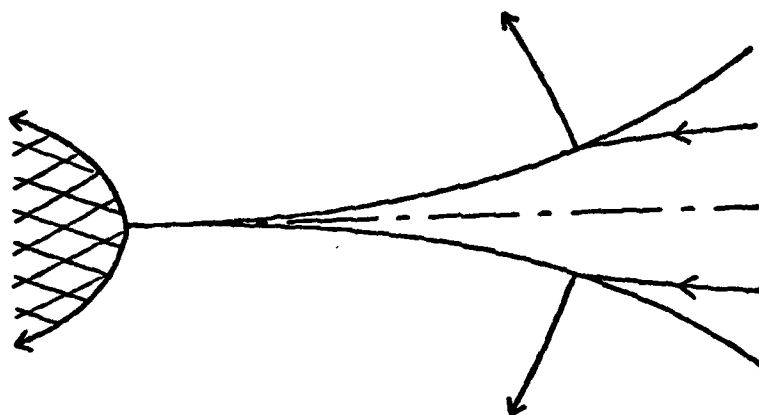


Figure 1a

In the present work, an accurate solution is found by a numerical calculation of the solution of (9) in which the appropriate singular behavior at π is imposed.

Numerical Solution

From equation (9) and result (8), a system of $N + 1$ equations is obtained for $y_i = y(x_i)$, where $x_i = (i-1)x_f/N$, $i = 1, 2, \dots, N+1$ and the end value x_f is less than but close to π . The principal-value integrals in (9) are approximated near their singularities by expanding $y(\eta)$ in a Taylor series, which produces ordinary integrals. All integrals are then approximated by simple quadrature and all derivatives by finite differences. In this way, the following system (12) is obtained. This system of nonlinear algebraic equations is solved numerically using the Newton-Raphson method.

$$\begin{aligned}
 y_1 &= 0 \\
 y_1^2 + \frac{x_f}{N} \left\{ \left(\sum_{j=2}^{i-2} + \sum_{j=i+2}^N \right) (y_j \cot[\frac{(j-1)x_f}{2N}]) + \sum_{j=2}^N y_j (\cot[\frac{(j+i-2)x_f}{2N}]) - \right. \\
 &\quad \left. 2 \cot[\frac{(j-1)x_f}{2N}] \right\} + \frac{y_{N+1}}{2} \left(\cot[\frac{(N-i+1)x_f}{2N}] + \cot[\frac{(N+i-1)x_f}{2N}] - \right. \\
 &\quad \left. 2 \cot \frac{x_f}{2} \right\} + (y_{i+1} - y_{i-1}) \left(2 + \frac{x_f}{N} \cot \frac{x_f}{2N} \right) - 2y_2 = 0 \\
 &\quad (i = 2, 3, \dots, N-1)
 \end{aligned} \tag{12}$$

$$y_N = -2 \ln(\pi - x_N)$$

$$y_{N+1} = -2 \ln(\pi - x_f) .$$

The last two equations in (12) represent the logarithmic singularity at π .

For this calculation, N was taken to be 27 and $x_f = 3.13$. Imposing the behavior (8) at x_f alone was found to yield unacceptable results; this occurred for all three values of N that were tested ($N = 9, 18, 27$).

Fixing y at the last two points according to (8) corrected this, however. Fixing the third-to-last value as well did not appreciably change the solution, which is shown in Figure 2. Interpolating between the points (x_1, y_1) and numerically integrating from 0 to x_1 gives the approximate shape of the wave front

$$\frac{1}{\gamma} \bar{F}_0(\eta) = \int_0^\eta y(x) dx, \quad \eta \in [0, \pi],$$

as illustrated in Figure 3. A smooth curve is fitted to the points $(x_1, \gamma^{-1} \bar{F}_0(x_1))$ and is then appropriately extended beyond the interval $[0, x_1]$ to generate the graph in Figure 4 of a portion of the complete flame profile.

Discussion of Results

The constant speed of propagation

$$v = \gamma^2 \int_0^\pi \cot \frac{x}{2} y(x) dx$$

may be computed from the numerical results obtained for $y(x)$. It is found that $v \approx 1.4\gamma^2$. The corresponding dimensional speed of the wrinkled flame may be shown to be

$$u_w = u_b \left(1 + \left(\frac{\rho_u}{\rho_b} - 1 \right)^2 \frac{1.4}{(2\pi)^2} \right),$$

where u_b is the speed of the undisturbed flame front relative to the burned gas. A typical value of the thermal expansion coefficient ρ_b/ρ_u is 0.2. Thus, the speed of the steady plane flame is amplified by a multiplicative factor of 1.6. This is in excellent agreement with recent experimental observations of large-scale flames propagating in thin spherical or hemispherical shells (Ivashchenko and Rumyantsev, 1978; Lind and Whitson, 1977). In these investigations, it was observed that for mixtures with markedly different burning velocities, the measured space velocity was 1.5 to 2 times the normal burning velocity measured in the laboratory. Also, the flame became rough as it expanded, having a "pebbled" appearance.

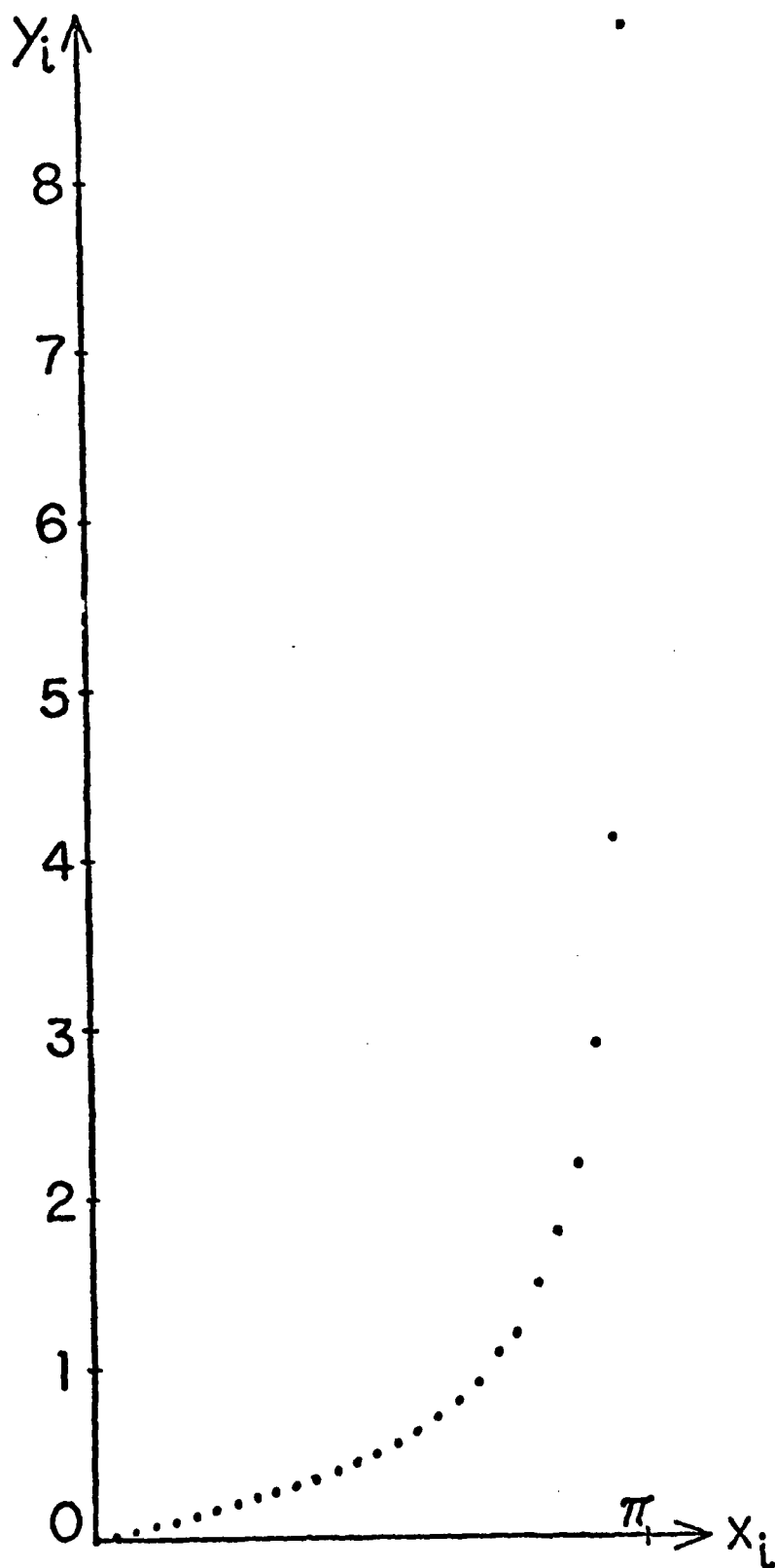


Figure 2. Plot of numerical solution of system (12).

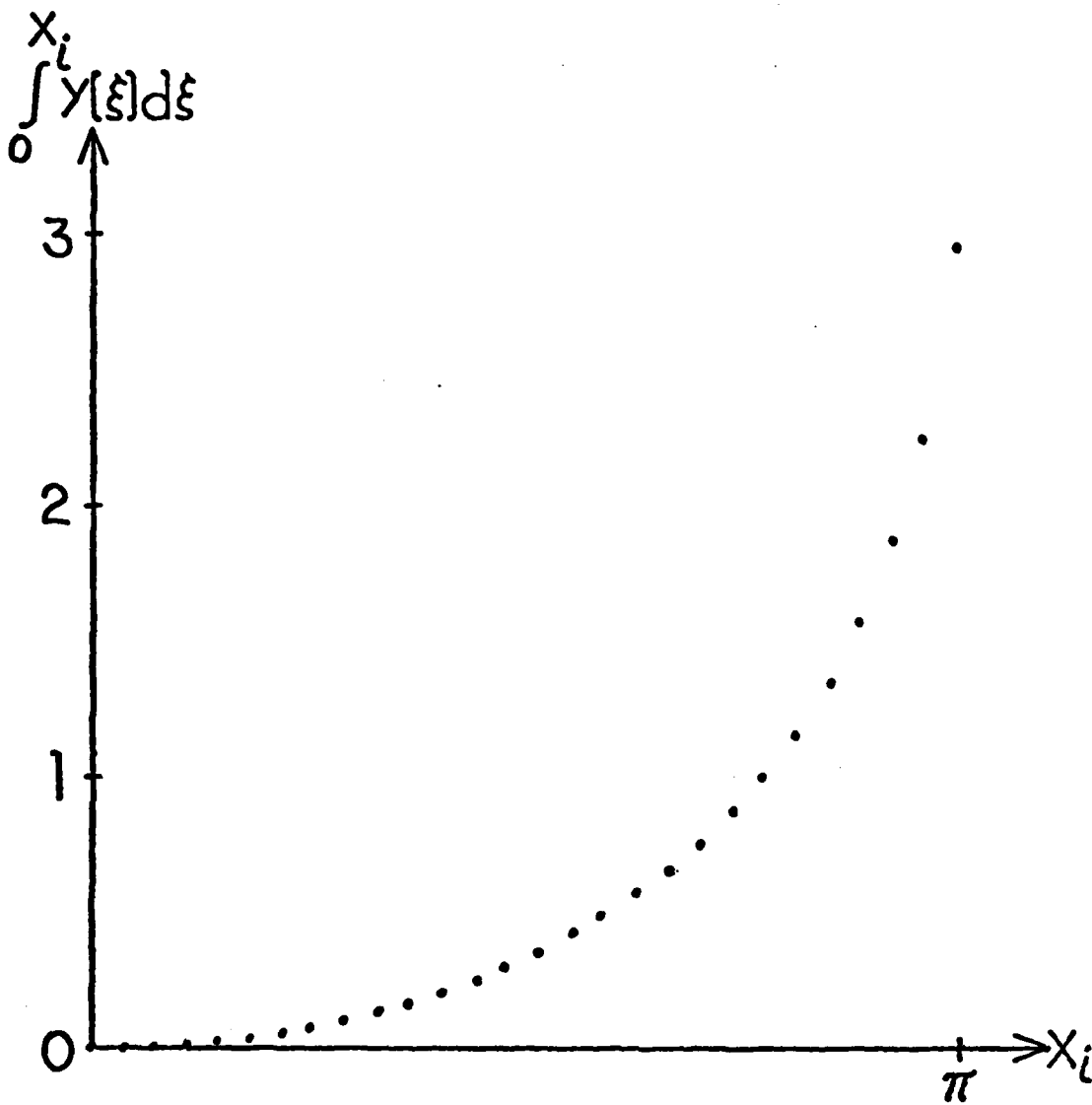


Figure 3

Approximate shape of the flame front $\gamma^{-1} \bar{F}_0(\eta)$ for $\eta \in [0, \pi)$.

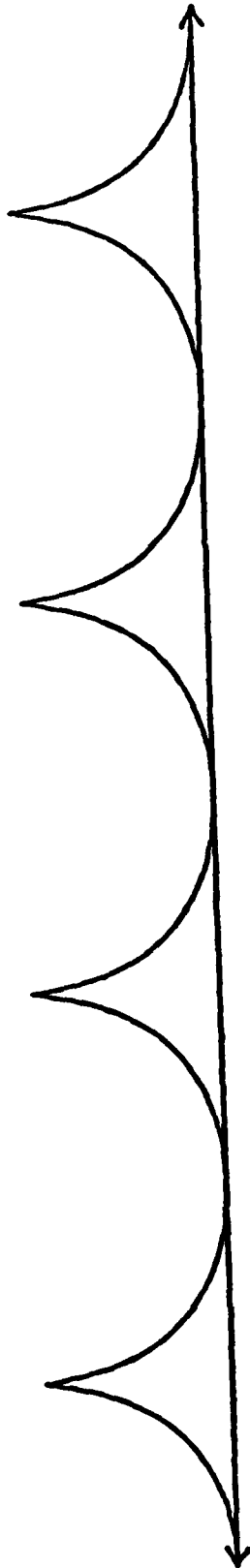


Figure 4

A portion of the complete flame profile.

Numerical investigation by Michelson and Sivashinsky (1977) of equation (3), which includes curvature effects, yielded the result

$$U_w = U_b \left\{ 1 + \left(1 - \frac{\rho_b}{\rho_u} \right)^2 (.18) \right\} .$$

For $\rho_b/\rho_u = .2$, this produces a magnification factor of only 1.1. This is apparently due to the narrowness of the integration interval taken. Over a relatively short interval, the stabilizing effect of the curvature term F_{0nn} may be significant, and so the wrinkles may be underdeveloped. Indeed, it is known from experiments on spherically expanding flames that the speed of a wrinkled flame stabilizes at an extremely low rate (Palm-Leis and Strehlow, 1969).

Another system to which the present findings may have some relation is flame propagation between two parallel walls (Uberoi, 1959). In particular, such a flame may be described by the curved front found here between two consecutive cusps. The propagation of a flame in a channel was recently considered by Zeldovich et al. (1980) where, however, estimates obtained both for the flame shape and for its propagation velocity were comparatively coarse. In that work, stagnation zones near the channel walls were taken into account, as shown in Figure 5.

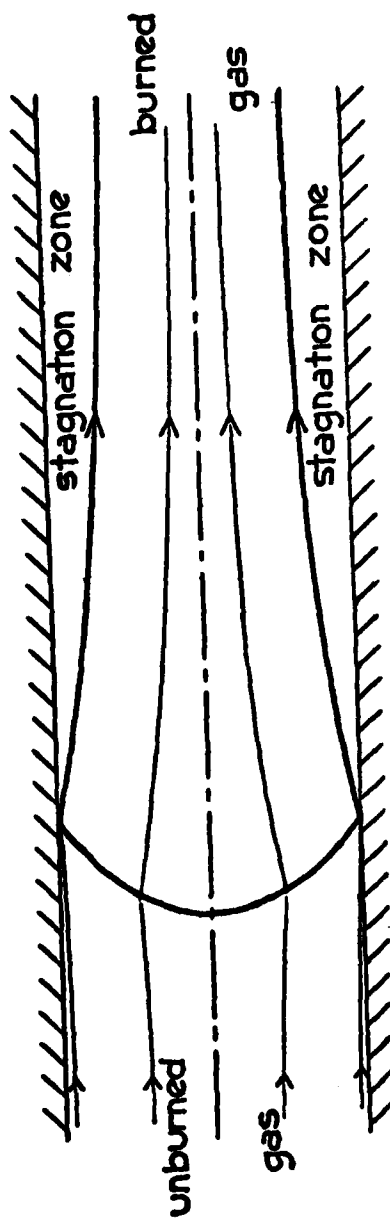


Figure 5

Flow associated with a flame propagating in a channel, as modeled by Zeldovich et al, 1980.

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